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TIME AND SPACE CAPTURE IN SPREAD SPECTRUM RANDOM ACCESS

I. Introduction and Summary

When a packet receiver is being accessed in a random access mode, the use of spread spectrum communication offers the possibility of time capture. That is, competing packets arriving during the transmission time of the first may be ignored if their signal strength is not too great. When the transmitters are widely distributed, geometric or power capture is possible [Roberts; 1972]. With or without spread spectrum, if a competing signal is much weaker (further away), it does not interfere. Both types of capture can give rise to performance superior to that predicted by the simple unslotted Aloha model. On the other hand, power capture in particular gives rise to bias against the more distant transmitters in the sense that the probability q(r) of a successful transmission at distance from the station will decrease as r increases, hence the number of retransmissions increase as well as delay.

It is the purpose of this report to characterize this degradation of service as a function of:

(1) the packet arrival rate,

(2) the time bandwidth product K(extent of spectrum spreading),

(3) the signal to noise ratio SN and of

(4) the exponent α in the inverse power law (S/r^{α}) assumed for the transmission power of a transmitter at distance r from the receiver.

The situation we consider is that of a circular field of transmitters of radius R accessing a receiver in the center where R is the range of the receiver. Arriving traffic is assumed to have a uniform rate per unit area in the field and the arrivals plus retransmissions are assumed to be Poisson distributed. Let S be the total arrival rate over the entire field; G be the total arrivals plus retransmissions rate; and G_u be the solution of $S = G_u e^{-G_u T}$, where T is the transmission time of a packet. It was found that $\frac{S}{G_u} \leq q(r)$. Morever, there is a critical radius $\bar{r} = \left(\frac{1}{1 + \frac{SN}{K}}\right)^{1/\alpha} R$ such that for $\bar{r} \leq r \leq R$ $q(r) = e^{-2GT}$ that is the system exhibits unslotted Aloha behavior in the interval (note however, the exponent involves G which is not known a priori) and q(r) is constant on this interval. As K or α increases, this interval becomes smaller and as SN increases, $\bar{r}$ decreases.

In Section 2, these and other results are derived after the model and the assumptions used are described in Section 2. In Section 4, simulation is used as a means of estimating accurately

G and for verifying the qualitative characterization of q(r) derived in Section 2.

The first analysis of the geometric effect of capture for a circular field of packet transmitters accessing a receiver in a random access mode was [Roberts; 1972] . Roberts made (among others) the following assumptions:

(1) the probability of a packet being received correctly q for a given transmission or retransmission is:

  (a) independent of r;

  (b) independent of whether the transmission is the first or a retransmission;

(2) the probability of more than one competing packet is negligible;

(3) noise is negligible compared to competing signal strengths.

Kleinrock and Lam [1972] improved on these results by relaxing assumption (1 b) to allow different q's for the initial transmission and for retransmissions. John Leung [1973] considered the same problem where the signal power, instead of being represented as a deterministic inverse square of distance, had a Rayleigh distribution to represent the effects of multipath. One consequence of this is that the receiver has unlimited radius. Leung also essentially assumes (1) and (2) (see in particular, eq. 7, where G(r) is apparently assumed proportional to r; i.e., that the rate of arrivals plus retransmissions per unit area is independent of r.

Fralick [1973] also does an analysis of the field of terminals situation, extending Roberts' results for the use of spread slotted spectrum, including propagation delays. He assumes (1), (2), and (3).

Abrahamson [1973] considers a central receiver in an infinite field of transmitters and determines the "Sisyphus distance" R_s which is the radius beyond which the expected number of retransmissions is infinite. He assumes (2) and (3). Thus, the critical distance R_s is defined by competing signals while the R we use is defined by noise considerations.

The model for spread spectrum reception is similar to those described in [Kaiser; 1973, p 2.] and [McGuire; 1973]. For simplicity, we assumed that the surface wave devices are programmable and that the chip code varies from bit to bit in a pseudo-random manner so that competing signals appear more nearly like noise; this, for example, avoids much of the difficulties pointed out by Fralick [1973]. Such devices are discussed in [Staples and Claiborne; 1973].

2. Model and Assumptions

A central receiver (station or repeater) is receiving messages from an infinite number of terminals in a radius R of the receiver where R is defined to be the range of the station; that is, the distance at which a terminal can just be heard. The terminals send packets according to a Poisson distribution with arrival rate density ρ packets per unit time per unit area. The principal objective is to determine the grade of service for terminals as a function of distance from the station. The grade of service Q(r) is defined to be the probability that a message sent by a terminal at radius r from the station will be received by the station on any one transmission (first and subsequent retransmissions are assumed to have the same probability of success [Kleinrock & Lam; 1972]).

Having q (r) we can also define G (r) = S (r)/q (r) to be the arrival plus retransmission rate at radius r when S (r) is the arrival rate at radius r given by $2\pi r\rho$.

For our purposes, the station can be in one of two states: free and busy. A free station is awaiting a synchronization code from the beginning of some packet. If it achieves synchronization, the packet captures the receiver which is now busy (in a receiving state) and remains in that state for one packet transmission time, T; during this time other arriving packets are lost (as well as the packet in synchronization if the competing signals have sufficient power). We specifically ignore the possibility of false alarms; that is, the receiver going from free to busy on

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type so that the competing signals can be lumped with the ambient noise. Suppose our receiver works by integrating the received signal correlated with the chip code. If the integral of the signal amplitude over one chip time is A and the noise integrated over one chip time has energy (variance) σ^2 the result of correlation and integration over K chips will be KA for the signal and a Gaussian noise distribution with variance $K\sigma^2$ for the noise. The ratio of the signal and noise powers is then

$$\frac{K^2 A^2}{K \sigma^2} = \frac{K S}{N}$$ i.e., by spectrum spreading, the

effective signal strength is increased by a factor of K. Similarly, the equivalent Gaussian contribution to the bit noise of a competing signal with amplitude chip integral of A^1 but uncorrelated with the code for the primary signal is a sum of K binomial variables which for K reasonably large is Gaussian with mean 0 and variance KA^{12} while the correlated signal gains by a factor of K^2. Thus, we will then assume that a bit is received correctly if

$$\frac{\frac{S}{R^\alpha}}{\frac{1}{K}\left(\sum_i \frac{S}{r_i^\alpha} + N\right)} \geq SN \qquad (2.1)$$

The range R is determined by

$$\frac{\frac{S}{R^\alpha}}{\frac{1}{K} N} = SN, \text{ or}$$

$$R = \left(\frac{K}{N} \frac{S}{SN}\right)^{1/\alpha} \qquad (2.2)$$

The parameters we wish to study are SN, α, and K; so, by judicious choice of units, we may normalize power and distance, so that s = 1 and R = 1. This results in the decision criterion:

$$\frac{K \cdot \frac{1}{r_0^{\alpha}}}{\sum \frac{1}{r_i^{\alpha}} + \left(\frac{K}{SN}\right)} \geq SN \qquad (2.3)$$

$$0 \leq r_i \leq 1 \qquad i = 0, 1, \ldots$$

See [Kaiser; 1973] for a similar model.

3. Preliminary Analysis

The exact analysis of the probability, $q(r)$, of a message getting through on any given transmission from a radius r seems most difficult; however, we can get quite a good qualitative idea of $q(r)$ by analysis. This analysis will be supplemented by simulation results in the next section.

Since the signal power is monotone decreasing with increasing radius and the message arrivals are independent, it is clear that $q(r)$ is monotone non-increasing. A message being transmitted, starting at time t from the critical radius $R = 1$, will be successful if, and only if, there is no transmission in the interval $(t-T, t+T)$ since any additional signal will prevent reception. But this is exactly the situation for unslotted Aloha. Thus, if G is the rate of message arrivals and retransmissions for the entire circular area of R radius from the receiver

$$q(R) = e^{-2GT} \qquad (3.1)$$

Unfortunately, we do not yet know G. Clearly from (3.1) and the fact that $q(r)$ is monotone $G \leq G_u$ where G_u is the arrival plus retransmission rate for an unslotted Aloha system; i.e., G_u is the solution of

$$S = G_u e^{-2G_u T}$$

where $S = \pi R^2 \rho$ is the total arrival rate.

We can generalize (3.1) somewhat. For even if $r < R = 1$, if r is sufficiently large, any other transmission can prevent it from being received. Thus, suppose we are considering a message from $r_1 = R = 1$, the weakest possible. The message can be heard whenever

$$\frac{K \frac{1}{r_0^a}}{1 + \frac{K}{SN}} \geq SN$$

or, $$r_0 \leq \left(\frac{1}{1 + \frac{SN}{K}}\right)^{1/a} = \bar{r} ;$$

On the other hand, if $r_0 \geq \bar{r}$, any other message is sufficient to block reception.

Therefore, we have

$$q(r) = e^{-2GT} \geq e^{-2G_u T} \qquad (3.2)$$
$$\text{for } \bar{r} \geq r \leq 1.$$

As $r \to \infty$, the signal power becomes infinite, so that the probability of successful transmission depends only on whether the receiver is free or not. Certainly the receiver is free if there has been no transmissions in the interval $(t-T, t)$ yielding

$$g(o) \geq e^{-2GT}.$$

Given there are one or more transmissions in $(t-T, t)$ the probability the receiver is captured is greater than the probability that one specific transmission, say the first in the interval, captures the receiver which is, in turn, greater than the probability that specific transmission is received which is S/G on the average.

Thus,

$q(0) = 1 - (\text{Prob of transmission in } (t-T, t). \text{ Prob receiver is captured by one of the transmissions})$

$\leq 1 - (1 - e^{-GT}) \frac{S}{G}$

yielding

$$e^{-GT} \leq q(0) \leq 1 - \left(1 - e^{-GT}\right) \frac{S}{G} \qquad (3.3)$$

Now let us summarize our knowledge about $q(r)$. We have:

(i) $e^{-2G_u T} \leq e^{-2GT} \leq q(r) \leq 1 - \left(1 - e^{-GT}\right) \frac{S}{G}$

for $0 \leq r \leq 1$

(ii) $q(r_1) \leq q(r_2)$ for $0 \leq r_1 \leq r_2 \leq 1$

(iii) $e^{-G_u T} \leq e^{-GT} \leq q(0) \leq 1 - \left(1 - e^{-GT}\right) \frac{S}{G}$

(iv) $q(r) = e^{-2GT}$ for $\left(\frac{1}{1 + \frac{SN}{K}}\right)^{1/\alpha} \leq r \leq 1$

where G_u satisfies $S = G_u e^{-G_u}$

4. A Simulation Approach.

Simulation was resorted to in order to verify the relations derived in Section 3 and to estimate more detailed attributes of the system in particular to estimate G which is required for several of the estimates in Section 3. Details of the simulation method are given in the Appendix. In general, the estimates of G were quite stable, with apparent accuracies of better than 1%. The functional form of q(c) was not as successfully simulated; however, the simulations were consistent with the approximations and bounds of Section 3 within sampling error.

The nominal system simulated had S = .1, SN = 20, K = 100, and $\alpha = 2$; T was everywhere taken to be 1. Table 4.1 summarizes the systems simulated and the results are given in Figures 4.1 to 4.7. In addition, in Figures 4.8 and 4.9 G as a function of K and S respectively are plotted.

System Number	Arrival Rate S	Signal to Noise SN	Spread Spectrum Factor K	Power Exponent α	Critical Radius $\bar{r}$	Measured Rate G
1	.1	10	100	2	.913	.1170
2	.2	20	100	2	.913	.2920
3	.1	10	100	2	.953	.1152
4	.1	20	10	2	.577	.1255
5	.1	20	1000	2	.990	.1126
6	.1	20	1	2	.223	.1290
7	.1	20	100	4	.955	.1166

Table 4.1
Simulation Runs

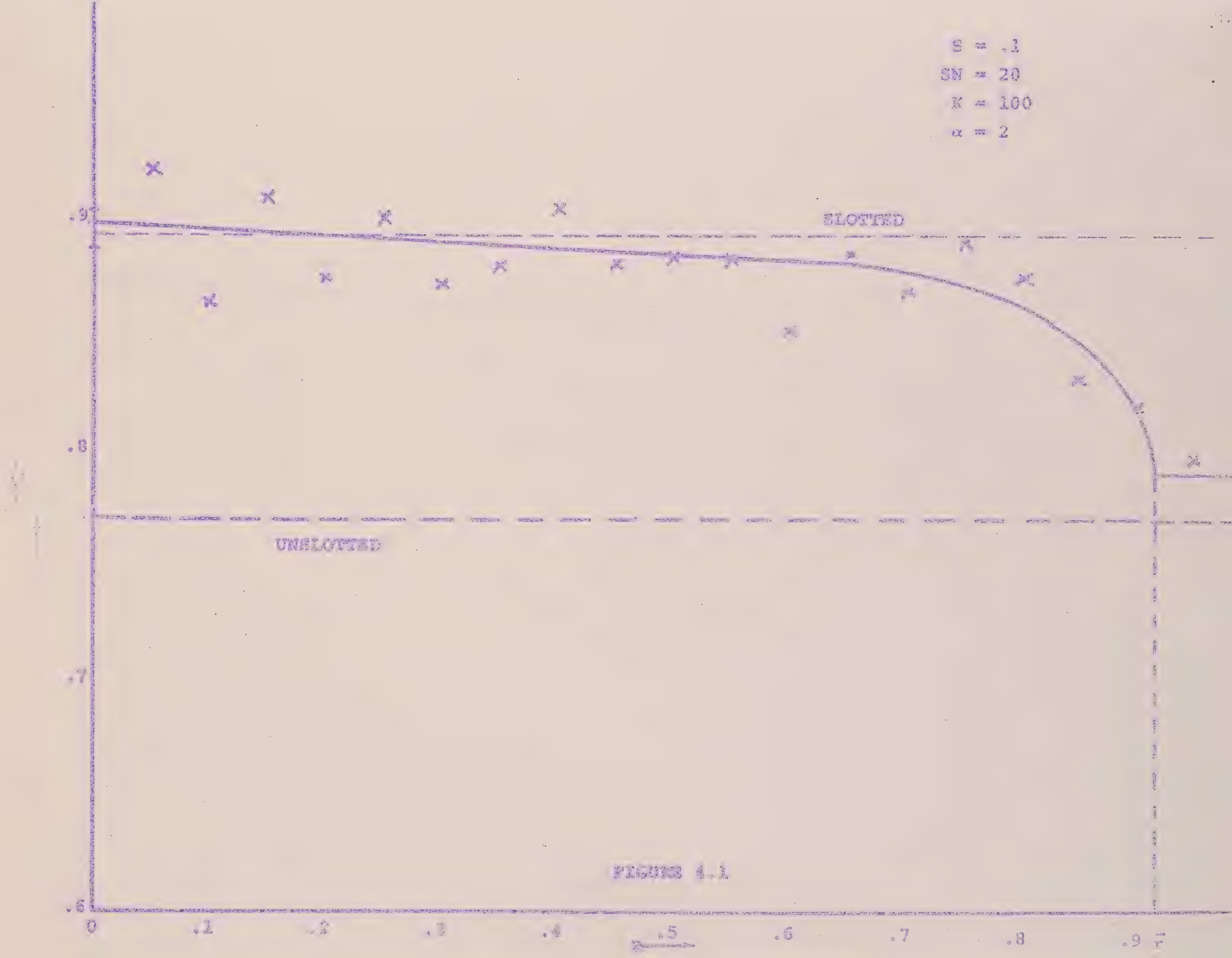


FIGURE 4.1

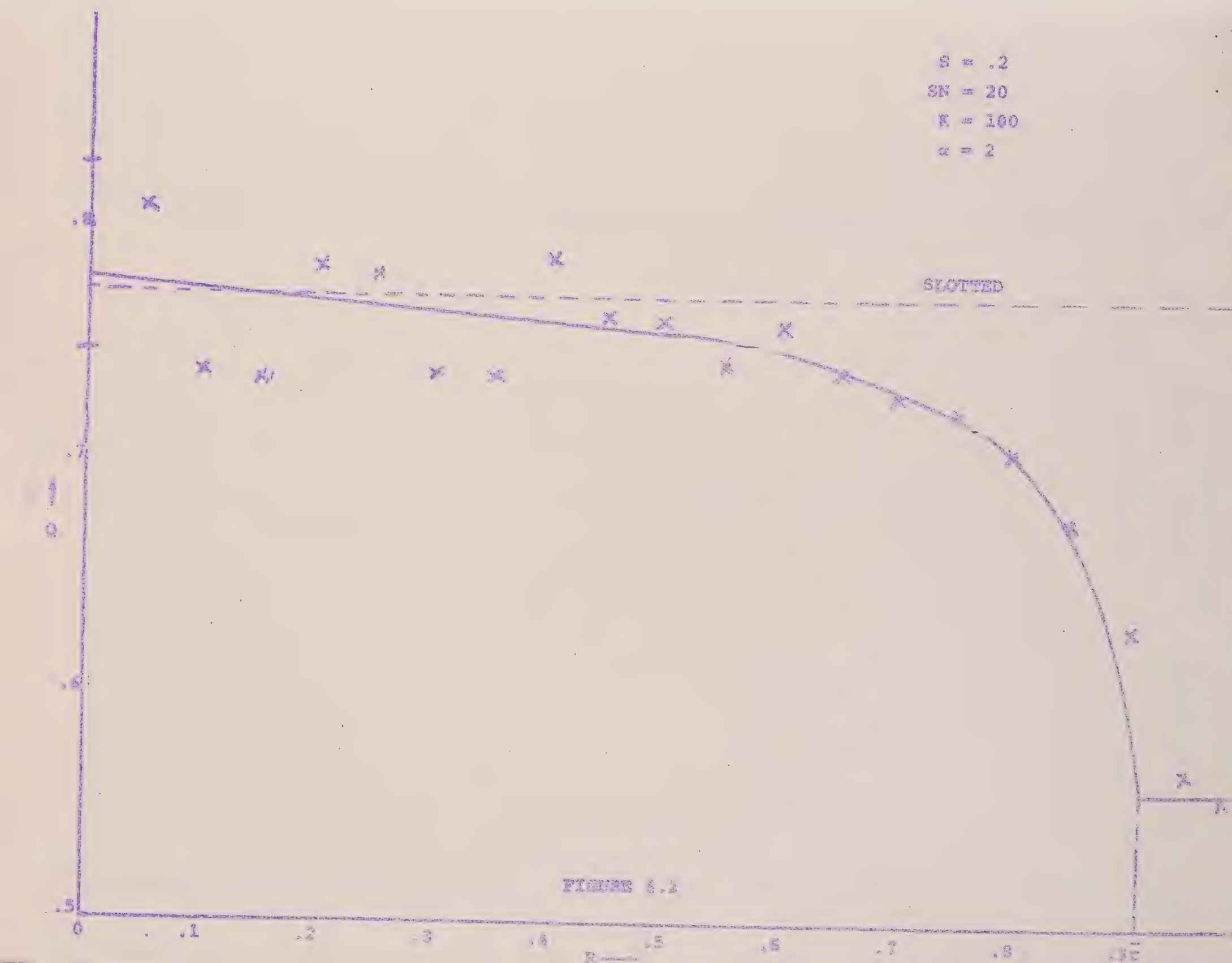


FIGURE 6.2

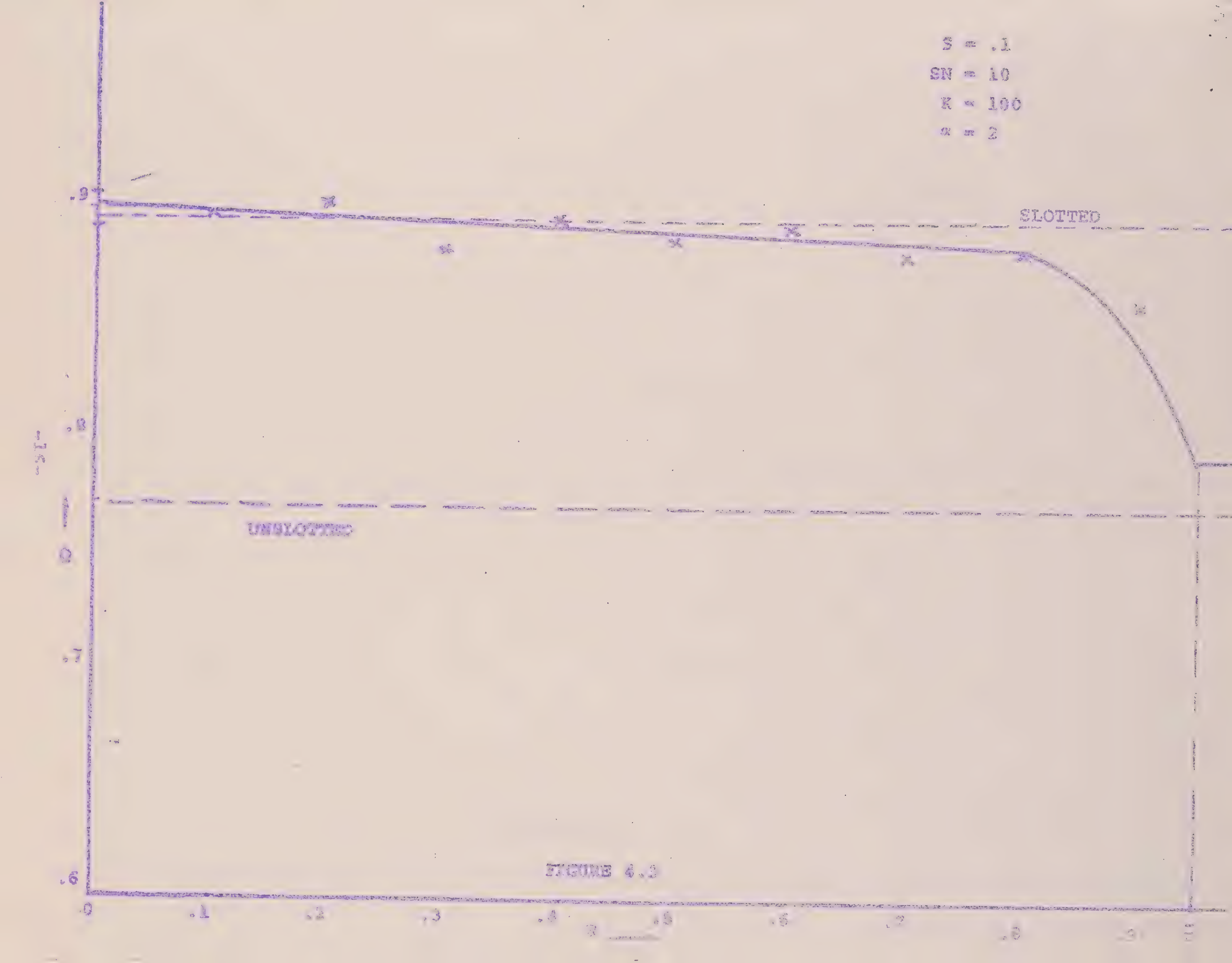


FIGURE 4.3

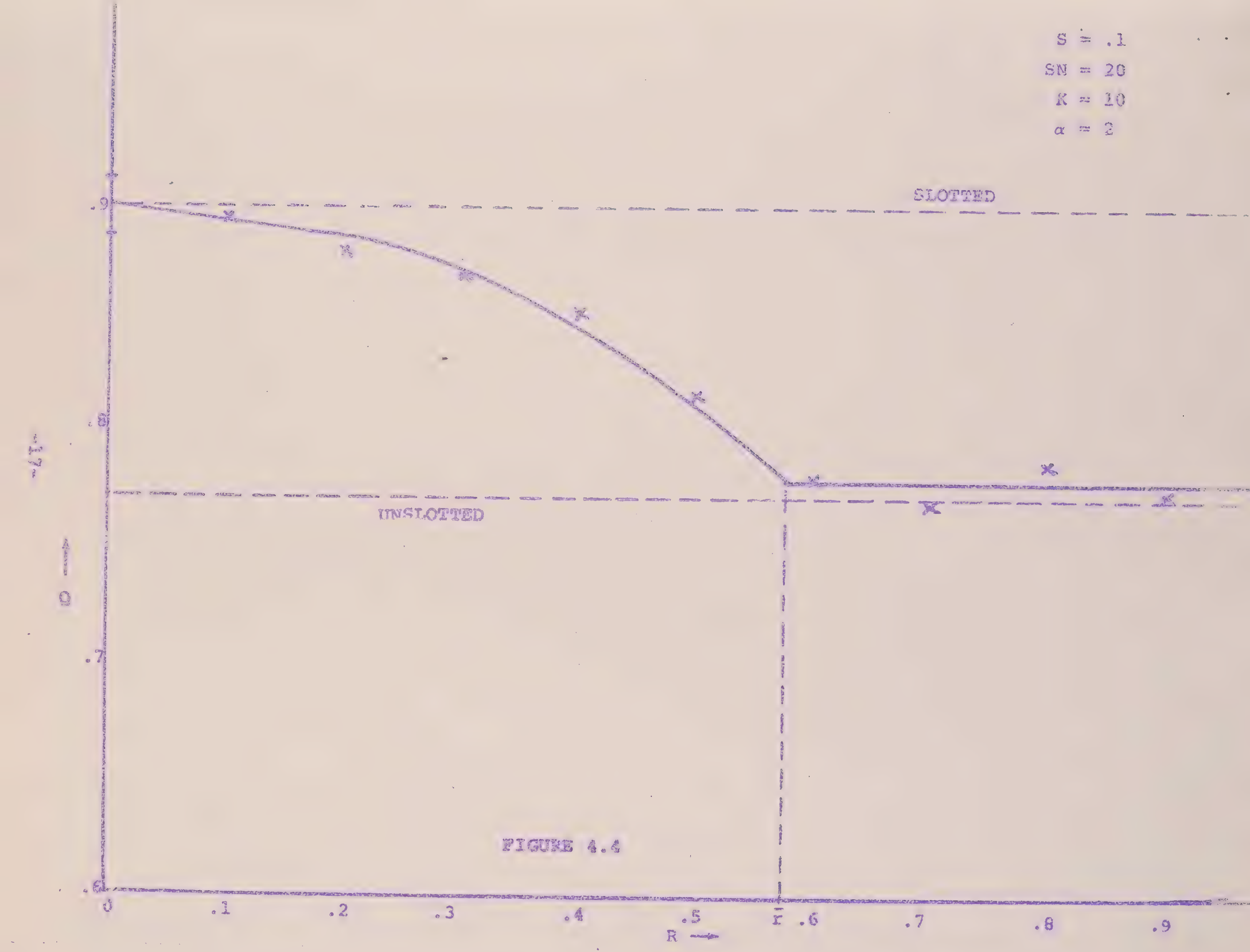


FIGURE 4.4

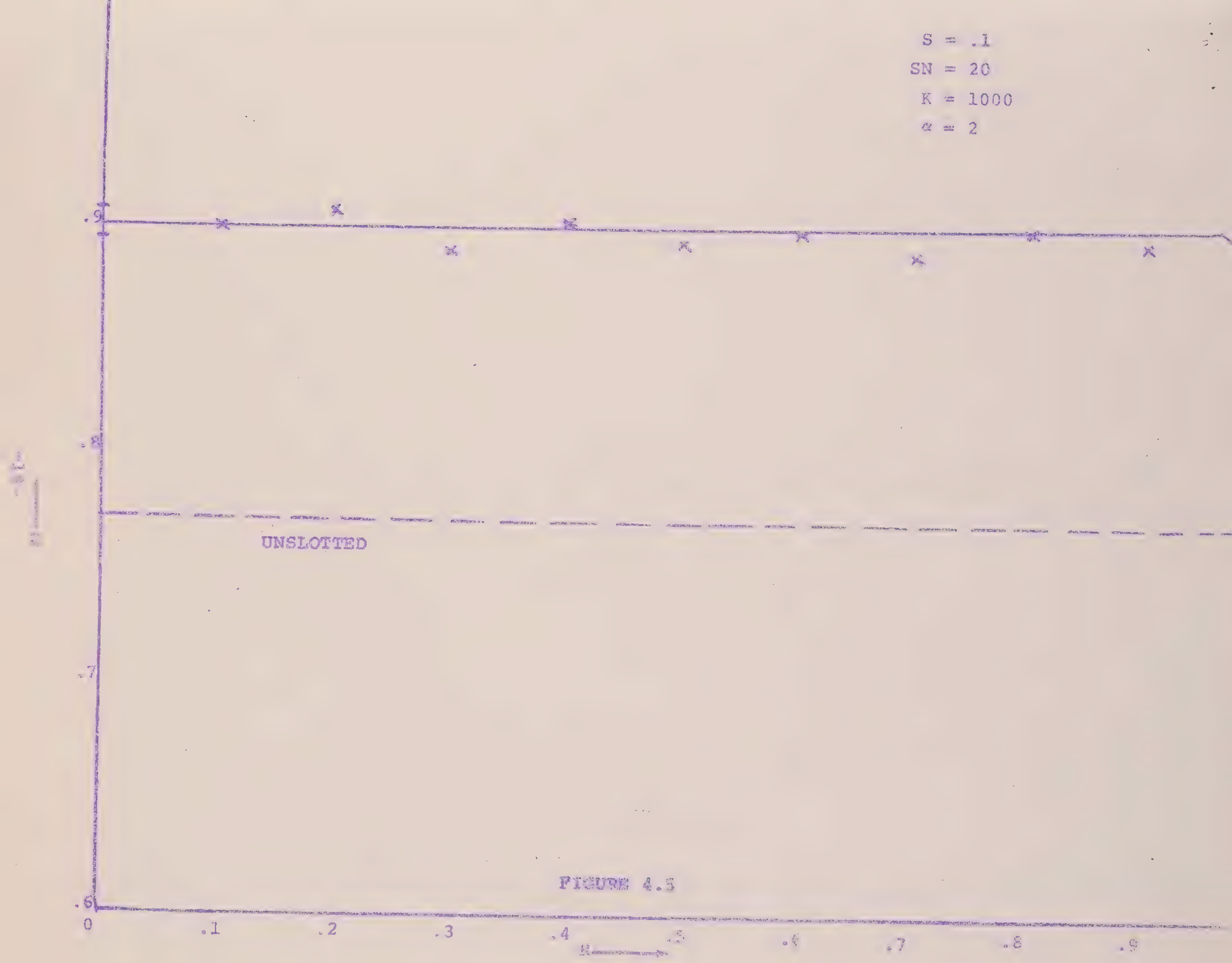


FIGURE 4.5

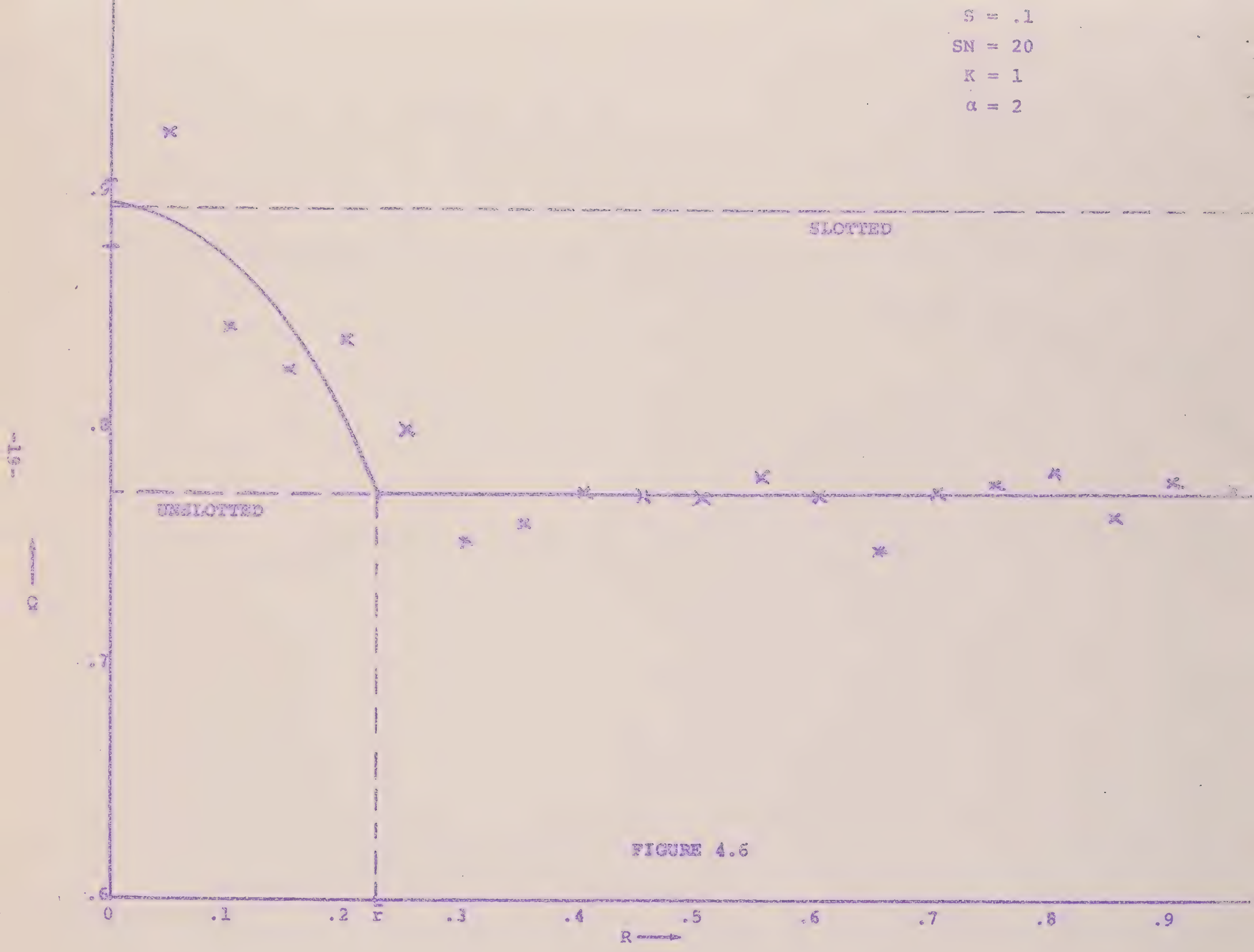


FIGURE 4.6

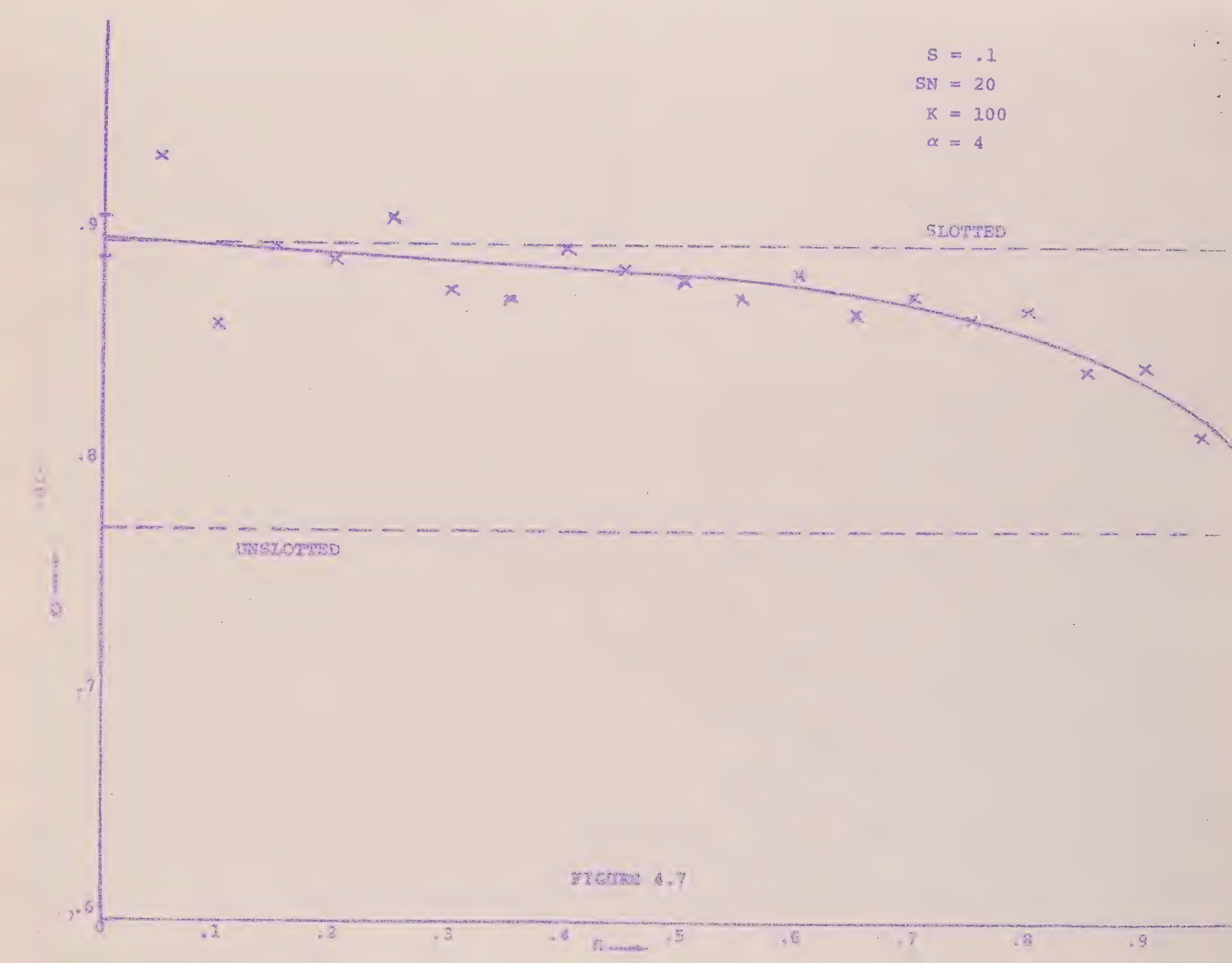
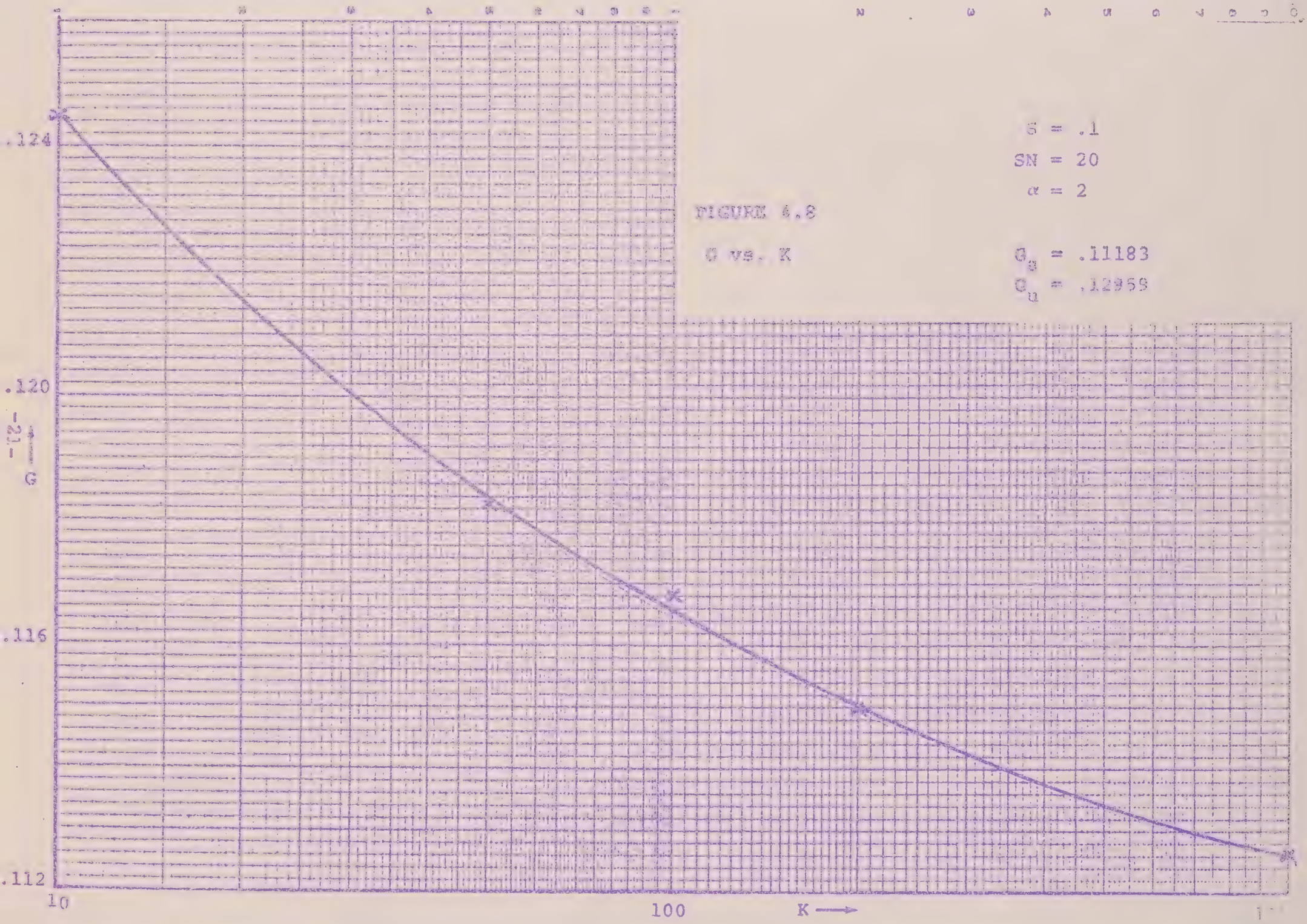


FIGURE 4.7

NO. 350-LOG DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH
EUGENE DIETZGEN CO.
MADE IN U.S.A.



S = .1

SN = 20

α = 2

FIGURE 4.8

G vs. K

G_s = .11183

G_u = .12959

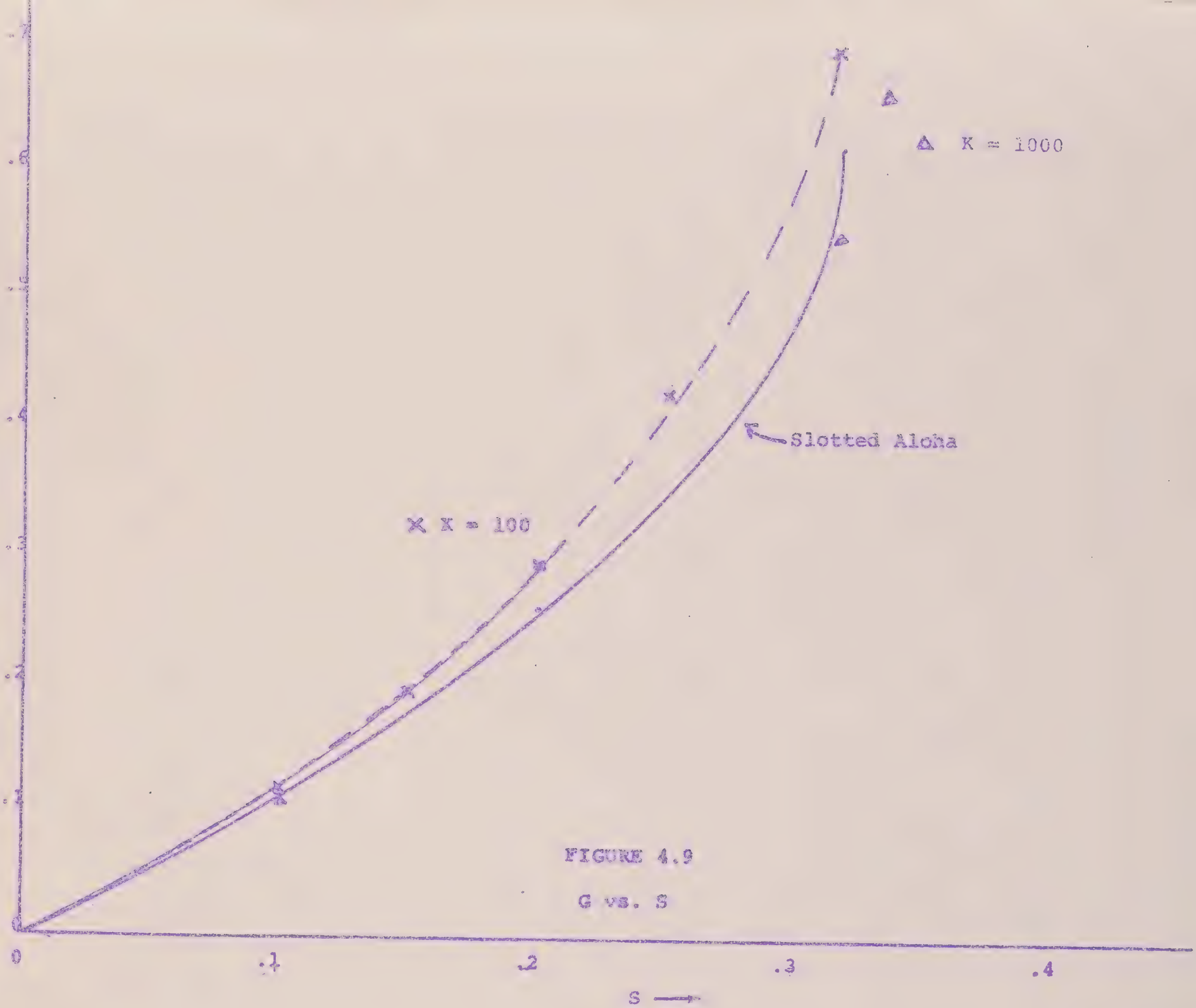


FIGURE 4.9

G vs. S

APPENDIX: Method of Simulation

The radius from the station to R = 1 is divided into ND intervals. Associated with each interval, I, is a ring of width I/ND and area 2π I/ND. Thus each ring is assigned a weight (proportional to radius)of arrivals in an array RDN, so that the arrival rate in the ring is RDN(I)$_*$ SIGMA; Q(I) is the current estimate of the probability a transmission from the I^{th} ring is successful. RDL (I) is defined proportionnal to RDN(I)/Q(I), so that the estimated arrival plus retransmission rate in the I^{th} interval is RDL(I) $*$ GAMMA, where SIGMA ~ S and GAMMA ~ G.

Initially, we take GAMMA = SIGMA and RDL = RDN. The general step is to generate NS Poisson arrivals with arrival rate GAMMA. The transmissions are assigned radii according to the distribution RDL and the corresponding signal powers calculated in an array S. In an array ST, the instantaneous power is calculated for each time at which a transmission is initiated. Finally, in an array SMX the maximum power over the T seconds subsequent to each start of transmission is calculated.

To calculate the number of successful transmissions, the transmissions are considered in order. An indicator keeps track of when the receiver is busy. For a given transmission, the program first determines which interval from 1 to ND the transmission originates from; if it is in the I^{th} for example, then RBX(I) is bumped. Then the program makes the following tests to see if the transmission is successful:

1) Using the ST array, the program determines if the receiver can hear the beginning of the transmission. If not, we go on to the next transmission, otherwise go to Step 2.

2) Is the receiver free? If it isn't, go on to the next transmission; if it is, mark the receiver busy for the next T seconds and go on to Step 3.

3) Can the entire transmission get through without being drowned out by subsequent competing transmissions? This is determined using the array SMX. If this test is passed, the transmission is assumed successful and a counter SBX(I) is bumped.

After all the transmissions are processed, an improved estimate for Q(I) given by

$$Q(I) = SBX(I)/RBX(I) \qquad (A.1)$$

is obtained and this is repeated several times until the distribution settles down. It turns out that this process was very unstable, at least in the estimate for Q(I). The estimates for GAMMA were quite easy to obtain and reliable, but there were rather wide fluctuations in the Q(I) between iterations or when using different seeds for the random number generator. The reason is apparently because the power for originations near the origin become unboundedly large and hence have disproportionate influence, while the number of such events is quite small so the resulting variance is rather large. This was ameliorated to some extent by using exponential smoothing; that is, replacing (A.1) with

$$Q(I) := RLX * Q(I) + (1-RLX)\ SBX(I)/RBX(I) \text{ where}$$

$$0 \leq RLX < 1.$$

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